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Two papers were completed during this time, one mathematical and one experimental on forced chains oscillators, motivated by problems concerning the lamprey CPG for undulatory locomotion. Work continued with K. Sivard on interpretation of experiment, and we are preparing a paper on significance of data from split bath experiments. In addition, work was started with Bard Ermentrout on the mathematics of chians of oscillators with coupling topology more complicated then nearest neighbor. This paper deals with patterns that form when long inhibitory connections are added to local excitatory connections. The work was motivated by patterns of movement that emerge in early development of vertebrates.

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Final Report

Work Accomplished on AFOSR Contract AFOSR–90–0017

Mathematical Problems Concerning Rhythmic Processes
N. Kopell, Principal Investigator
9/1/89 – 8/31/90

A-1

Work was accomplished on two categories of problems. The first concerned questions directly related to Central Pattern Generators, as discussed in the proposal for this contract. Two papers were completed during this time, one mathematical and one experimental on forced chains oscillators, motivated by problems concerning the lamprey CPG for undulatory locomotion [1,2]. Work continued with K. Sigvard on interpretation of experiment, and we are preparing a paper on significance of data from split bath experiments [3]. In addition, work was started with Bard Ermentrout on the mathematics of chains of oscillators with coupling topology more complicated than nearest neighbor [4]. This paper deals with patterns that form when long inhibitory connections are added to local excitatory connections. The work was motivated by patterns of movement that emerge in early development of vertebrates.

Other work has been started on problems related to the properties of small neural networks. The focus of that work is to relate the emergent behavior of the network to general biophysical properties of the individual cells For example, one student is now exploring how the existence of slow ionic currents that turn on when the cells are inhibited can help to coordinate different subnetworks (pyloric and gastric) of the crustacean stomatogastric ganglion operating on different time scales. Mathemtically, the work concerns the forcing of an excitable system by an oscillatory one; the desired results are sub and super–harmonic solutions with fixed phase relationships between marker events in the forced system and the forcing oscillator. Regular meetings have been held with Prof. E. Marder and her collaborators Prof. L. Abbott and Dr. T. Kepler to discuss the formulation of mathematical problems on the emergent behavior of small networks of (wet) neurons.

Work is now being completed with a student D. Somers on a problem motivated by phenomena observed in the visual cortex, but also relevant to behavior of the lamprey spinal cord. [5] The question is whether local (e.g. nearest neighbor) architecture is sufficient to allow rapid phase–locking in a chain of oscillators. Our simulations show that oscillators of relaxation type can have a dramatically different transient behavior, with a much smaller time necessary to reach coherence. Relaxation oscillators are also less vulnerable to "oscillator death", or cessation of rhythm due to mutual strong coupling by impulses. In addition to the

simulations based on well–known neural models, this paper contains mathematical caricatures used to explain the behavior of the system. We also show that the complicated models numerically show some features not previously reported, such as "fractured synchrony", in which there are domains with synchrony intermixed with other domains synchronized at a half–cycle distance, and "popout", or the emergence of synchrony in a few cycles after many cycles of no coherence.

The second category of work was outside the explicitly proposed tasks, though it was partially motivated by the desire to build up technique to handle the problems of singularly perturbed systems that arise in dealing with coupled relaxation oscillators or excitable media with different time scales. Two problems dealing with singularly perturbed systems were solved. The first, worked on in collaboration with C. Jones, concerns techniques for following the inclination of invariant manifolds as they pass close to the "slow manifold". The key result, called the "exchange lemma", describes the position of the tangent to such an invariant manifold as it leaves a neighborhood of a slow manifold. The lemma can be used, for example, to turn singular solutions of travelling wave equations into locally unique actual solutions for small values of the small parameter. This work was described in two conference prodeedings [6,7] and was reported on in a plenary talk at the SIAM Meeting on Dynamical Systems at Orlando. The full paper is near completion [8].

The second work, done with Mike Landman, continues the theme of how geometry can be used to illuminate asymptotics and turn approximate calculations into rigorous proofs. This work deals with the second order (complex), nonlinear and nonautonomous system derived by Papanicolao and his coworkers to describe the profile of some solutions to the nonlinear Schrodinger equation as they approaches blowup. Asymptotic calculations are used to help construct a pair of invariant manifolds and to show that there is a transversal intersection which is the desired solution. Techniques include many changes of variables motivated by the asymptotics. This work is currently being written up [9].

[1]. T. Williams, K. Sigvardt, N. Kopell, G.B. Ermentrout and M. Remler, "Forcing of coupled nonlinear oscillators: studies of intersegmental coordination in the lamprey locomotor central pattern generator", Journal of Neurophysiology, 64, 862–871, 1990.

[2]. N. Kopell, G.B. Ermentrout. "On chains of neural oscillators forced at one end", to appear in SIAM J. Appl. Math.

[3]. K. Sigvardt, N. Kopell and G.B. Ermentrout, "Split bath preparations reveal aspects of intersegmental coordination of lamprey central pattern generator for locomotion", in preparation.

[4]. G.B. Ermentrout and N. Kopell, "Long–range interactions in chains of neural oscillators", in preparation

[5]. D. Somers and N. Kopell, "Contrasting properties of coupled relaxation and non–relaxation oscillators", in preparation.

[6]. C. Jones, N. Kopell and R. Langer, "Construction of the FitzHugh–Nagumo pulse using differential forms", to appear in Pattern and Dynamics in Reactive Media, IMA Volumes in Mathematics and its Applications, H. Swinney, G. Aris, and D. Aronson, eds, Springer–Verlag.

[7]. N. Kopell, "Dynamical Systems and the Geometry of Singularly Perturbed Equations", to appear in the Proceedings of the Smalefest (1990), Springer–Verlag.

[8]. C. Jones and N. Kopell, "Tracking invariant manifolds with differential forms", in preparation.

[9]. N. Kopell and M. Landman, "A geometric view of blow–up in the nonlinear Schrodinger equation", in preparation